Speeding up the Computation of the Interpolation Operator in an Algebraic Multigrid Solver
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Goal: Speed up the calculation of eigenvectors that are used in the computation of an interpolation operator in a spectral, two grid algebraic multigrid solver.

Two-grid Adaptive Smoothed Aggregation Spectral AMG Method

1. Pre-smoothing: \( y = x_i + M^{-1}(b - Ax_i) \)
2. Coarse grid correction
   a. restrict the residual: \( r_c = P^T (b - Ay) \)
   b. coarse grid equation: \( A_c x_c = r_c \)
   c. update fine grid iterate: \( z = y + Px_c \)
3. Post-smoothing: \( x_{i+1} = z + M^{-T}(b - Az) \)

The Interpolation Operator (P)

- Agglomerates of finite elements, aggregates of vertices
- \( P \): Chebyshev-based polynomial; sparse; requires eigenvectors of agglomerates
- \( M \) (smoother): Chebyshev-based polynomial; sparse
- Parameters for AMG code:
  - elements per agglomerate (e)
  - degrees of polynomials for \( M \) and \( P \) (v)
  - spectral tolerance (θ)

Increasing the number of elements per agglomerate leads to a smaller but denser coarse grid, reduces the number of AMG cycles, but with a much costlier AMG hierarchy construction.

\[ \tilde{P}_i = [q_1, q_2, q_3, \ldots] \]

is obtained by solving an eigenvalue problem of the form

\[ A_T q = \lambda D_T q, \]

where \( A_T \) is the local stiffness matrix and \( D_T \) is a diagonal matrix (weighted smoother obtained from \( A_T \); it can be singular)

Original approach: \( A_T q = \lambda D_T q \Rightarrow D_T q = \frac{1}{1+\lambda} (A_T + D_T)q \Rightarrow \) LAPACK’s dsygvx

Iterative eigensolvers for \( A_T q = \lambda D_T q \):
- JADAMILU³: Jacobi-Davidson, different ways of setting the preconditioner
- ARPACK⁴ + SuperLU⁵ (shift-invert)

Case Study: SPE10 Benchmark (model 2)⁶

- Model of a formation in the Brent oil field; 1200’x2200’x170’ (fine scale cell size is 20’x10’x2’). The top 70 ft (35 layers) represents the Tarbert formation; the bottom 100 ft (50 layers) represents Upper Ness (fluvial).
- Modelled described by Brinkman equations⁷

References